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This grant was used for upgrading an SGI Power Challenge L supercomputer and for the acquisition of 3 SGI 02 workstations to serve DoD research projects in three general areas:

1. Research on the p- and hp-versions of the Finite Element Method conducted by the Center for Computational Mechanics.

2. Research on non-linear control systems conducted in the Department of Systems Science and Mathematics.

3. Rotorcraft analysis and simulation conducted in the Department of Mechanical Engineering.

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High Performance Computing for Engineering Analysis and Design

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The research supported by the computing equipment acquired through this Defense University Research Instrumentation Program (DURIP, FY97) equipment grant spans broad areas ranging from nonlinear control systems and finite element analysis to rotorcraft analysis, design and control.

The research requires extensive computer simulations previously performed on limited computer resources. The equipment acquired has impacted seven projects by providing much needed computing power. It also provided to the PI's individual computational capabilities which allows them to have direct access, through networking, to the central computer laboratory. Details are given in the following sections.

1 Equipment acquisitions

Three Silicon Graphics O2 workstations were acquired and a Silicon Graphics Power Challenge L server was upgraded to 448 MB RAM, 18 GB disk storage as user space, four 196 MHZ IP25 R10000 processors, an internal 5 GB 8mm tape drive, providing a a tightly coupled, coherent, shared memory multiprocessor supercomputer, located in the Laboratory for Computation and Control, on the Washington University campus.

2 Research projects

Brief summaries of the research projects that benefited from this DURIP equipment grant are summarized in the following.

2.1 Research on the p- and hp-Versions of the Finite Element Method AFOSR Grant No. F49620-95-1-0196, Barna A. Szabó, PI

Failure initiation and crack propagation are inherently nonlinear processes. Nevertheless, computations based on the linear theory of elasticity and the deformation theory of plasticity,

coupled with experimentation, have proven to be useful for predicting critical crack sizes and crack propagation rates in metals. Similar procedures are expected to be applicable to failure initiation in composite materials where the critical points are typically like those indicated in Fig. 1. In many important cases the primary loading is by temperature changes.

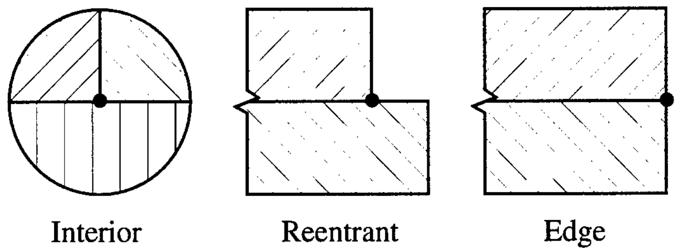

Figure 1: Typical singular points associated with multi-material interfaces.

The principal objective of this work is to develop analytical procedures that will make it possible to correlate failure initiation events with mechanical and thermal loadings in composite materials. There are many applications such as, for example, electronic packaging, ceramics, adhesively bonded joints and laminated composites. The goal is to replace expensive and unreliable trial-and-error methods which are currently used by providing a rigorous quantitative description of the natural straining modes at singular points. This project is a continuation of work performed under AFOSR/ARPA funding (Grant No. F49620-93-1-0173, Advanced Mathematical Models for Structural Systems). The formulation, based on a new method, called the modified Šteklov method, has been completed for elastostatic problems in a two-dimensional setting under the AFOSR/ARPA project. Work is being performed on the formulation of the thermoelastic problem. The solution in the neighborhood of critical points is characterized by eigenpairs, which represent the natural straining modes of the elastostatic problem, and the generalized stress intensity factor which quantifies the energy residing in each of the natural straining modes.

Therefore failure initiation criteria involve the eigenpairs and the generalized stress intensity factors either directly or indirectly. The eigenpairs are computed from the condition that they must satisfy the equations of anisotropic and inhomogeneous elasticity locally and satisfy the local boundary conditions.

The formulation is based on a generalization of the "Steklov problem". The formulation is finite element-oriented and uses special features of p-extensions. Numerical experiments have shown that the method performs very well. Several representative tests cases have been solved. The generalized stress intensity factors are computed by a new method based on the complementary energy formulation. The major advantages of the new method are as follows:

1. The computed generalized stress intensity factors exhibit superconvergence.
2. The method is applicable to anisotropic materials, and any type of singularities.

3. The method can be used in conjunction with any finite element analysis program.

There are two essential elements of successful failure initiation and damage tolerance analysis (DTA): First, a hypothesis concerning the relationship between certain parameters of the stress field and an observed failure initiation or crack propagation event. Second, the availability of convincing experimental confirmation that the hypothesis holds independently of variations in geometric attributes, loading and constraints. Having a reliable capability for analyzing the stress field in critical regions is of primary importance because failure hypotheses incorporate certain parameters of the stress field which *cannot be measured directly.* Progress is made through correlation of experimental results with computed information. Unless the computed information is reliable, and more accurate than the experiment, the development of a valid failure theory would not be possible.

Not having reliable theories of failure is very expensive to DoD and industry: Extensive testing programs are required which are very time consuming and generally lead to feasible but not optimal designs.

An opportunity exists for the introduction of new numerical simulation technology which will remove this barrier. This technology will make it possible to replace experimentation with numerical simulation, leading to better engineering decisions in less time and at a significantly lower cost.

It is essential to formulate failure criteria in terms of functionals the exact values of which are finite and not sensitive to minor variations in topology. The stresses corresponding to the exact solution are infinity in singular points and are very sensitive to topological changes, such as changes in fillet radii, therefore are not useful for formulating failure criteria. Failure initiation criteria must be based on coefficients which are analogous to the stress intensity factors in fracture mechanics, called generalized stress intensity factors (GSIFs).

The computation of generalized stress intensity factors from finite element solutions is one of the main areas of concentration of our current research.

Singular points are those points in a structural component where a reentrant corner occurs (like cracks and V-Notches), material properties abruptly change along a free edge, interior points of three (or more) zones of different materials intersect, or an abrupt change in boundary conditions occurs. Under the assumption of small strains, and using the linear theory of elasticity, failure theories and a successful DTA can be developed if the asymptotic solution in the vicinity of these singular points is known. The asymptotic solution is typically characterized by a sequence of eigenpairs. The determination of these eigenpairs, and reliable computation of the coefficients of the asymptotic expansion (GSIFs), constitute the scientific basis for the development of reliable DTA.

Numerical accuracy is essential because unless the accuracy of the computed data is known it would not be possible to tell whether the working hypothesis is wrong or the numerical errors are too large, or both. In some cases large errors in a working hypothesis are nearly

cancelled by similarly large numerical errors, leading to false conclusions.

When a heterogeneous mechanical object or structure is subjected to mechanical and thermal loading then singularities in the temperature distribution must be considered also.

In the neighborhood of singularities, caused (for example) by changes in material properties, the temperature distribution is characterized by functions coefficients which are analogous to the function sand coefficients (generalized stress intensity factors) of the elasticity problem. Accurate determination of these coefficients is an essential requirement because the thermal loading in the most critical region is characterized by these coefficients.

Other activities include the development of advanced models for structural connections. The models account for nonlinear effects, such as mechanical contact, material and geometric nonlinearities. A typical configuration is shown in Fig. 2.

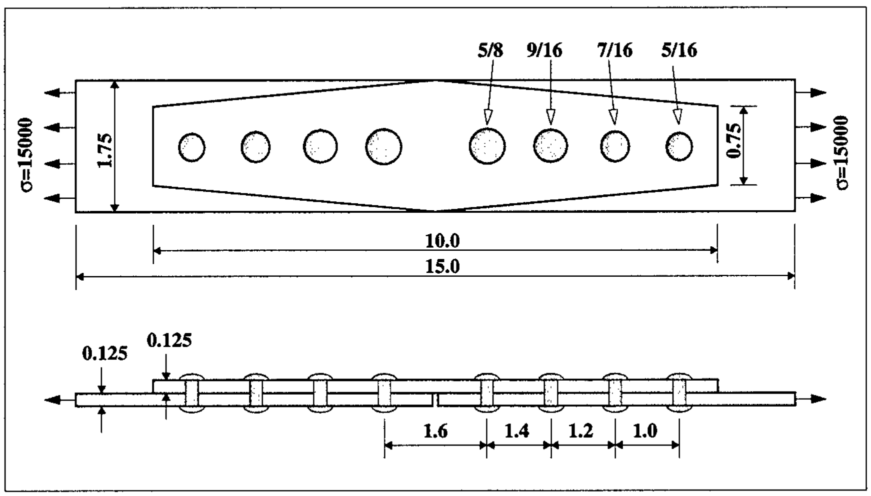

Figure 2: A model problem for fastened structural connections (the dimensions are in inches)

Efficient computation of margins of safety for critical military and civilian aircraft components was another area of research. A "bulky" aircraft component, requiring fully three-dimensional finite element analysis is shown in Fig. 3.

2.2 Nonlinear Control Systems AFOSR Grant No. F49620-95-1-0232 Christopher I. Byrnes and Alberto Isidori, PI

This research effort is aimed at the development of a control methodology for lumped and distributed parameter systems, similar in scope and applicability to classical automatic control design for lumped linear systems. The focus is on the development of geometric methodologies for systematic robust nonlinear feedback control and the initiation of a related investigation into systematic methods for taking advantage of genuinely nonlinear phenomena,

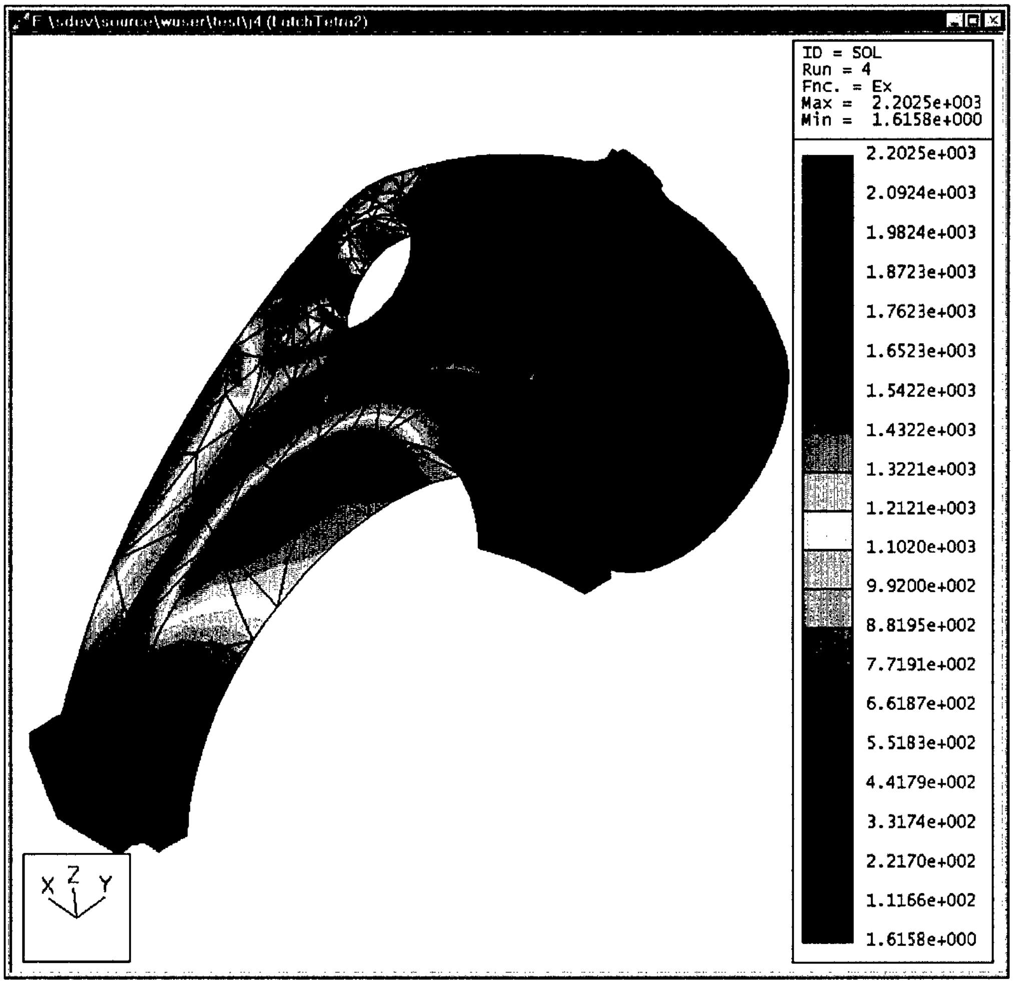

Figure 3: A "bulky" aircraft component requiring fully three-dimensional FEA analysis. Equivalent stress contours.

using a combination of methods drawn from nonlinear dynamics and geometric nonlinear control.

In this work there are two closely allied themes which have their origin in two approaches to the control of complex dynamical systems - the analysis and control of as accurate a system description as possible, and the alternative development of reduced order models, which capture some of the dominant dynamical effects and require the consequent development of high performance controllers which either are more robust with respect to unmodeled dynamics or which can adapt to unknown parameter fluctuations in the reduced order model.

Our specific research objectives are divided into two main categories: Lumped and distributed nonlinear control. First we will discuss our objectives for lumped systems.

2.2.1 Methodologies for Control System Design for Lumped Nonlinear Systems

Among the most important open problems in lumped nonlinear control systems are the development of a systematic theory of robust control and a better understanding of underlying mechanisms for taking advantage of nonlinear effects to shape the response of complex systems. As a starting point, we recall that one of the classical problems which involves shaping the response of a system is output regulation, in which the objective is that of controlling a plant in order to have its output track (or reject) exogenous commands or disturbances. In this direction, the nonlinear regulator theory developed in [Byrnes, Isidori, IEEE Trans. Autom. Control, 1990] is valid in case a model for an "exosystem" which generates the exogenous signal to be tracked and the exogenous disturbance to be rejected is known. The "exosystem" is also assumed to be neutrally stable, which includes many signal generators of practical interest (e.g., set-point control, oscillators and almost periodic motion) but does not allow for certain signal generators which also have application, such as tracking a ramp or a limit cycle. One of the goals of the present research effort is to extend the class of signal generators to which nonlinear regulator theory will apply.

Our preliminary calculation indicate that the incorporation of system immersion into geometric output regulator theory will allow us to give a definitive solution to our first two explicit research tasks:

Task 1: The development of sharper condition soft nonlinear output regulation with measurements

Task 2: Output regulation in the case of exosystems which are not necessarily "neutrally stable", including, for example, exosystems which have stable limit cycles, invariant tori, etc.

Among the additional research tasks we have proposed in this area, we consider the development of robust control schemes for output regulation. Even for lumped linear systems, the more refined task of developing a robust regulator theory involves still more challenging cases arising when either (a) a signal generator is known but certain parameters in the system or in the disturbance channel are unknown (structured uncertainty) or (b) a signal generator for disturbances is not known (unstructured uncertainty).

Both of these problems are central focal points in the modern approach to robust control. Despite significant effort, for lumped, linear systems robust control for structured uncertainties is not nearly as well understood as robust control for unstructured uncertainties. Nonetheless, the design of control systems which are robust against structured uncertainties is of clear importance to many DoD objectives, such as the development of flight controllers for high angle-of-attack or highly agile aircraft which are robust against nonlinear variations in the coefficient of lift or nonlinear couplings between wind axis moments and rectilinear moments. Our analysis of this research objective is stated in the following task.

Task 3: The development of a feedback design methodology for achieving robust regulation

in the presence of structured perturbations.

The starting point of our research effort in this direction is the proposed development of a reduction principle for the Isaacs equation, similar to the reduction principle of center manifold theory for nonlinear dynamical systems.

Task 4: The development of a reduction principle for the Hamilton-Jacobi inequality arising in nonlinear disturbance attenuation.

Since the application of such a reduction principle will also ultimately involve an analysis of low dimension problems, we are also investigating methods for the solution of planar disturbance attenuation problems.

Task 5: A complete analysis of problems of disturbance attenuation for nonlinear systems evolving in the plane.

Another problem addressed in this research effort is the existence of smooth solutions of the Isaacs equation. Standard approaches to solving this problem require the equilibrium point of a suitable Hamiltonian system to be hyperbolic. We have observed [A. Isidori, Proc. of 1st European control Conf 1991], however, that smooth solutions of the Isaacs equation may still exist and yield the required stabilizing properties even in case the equilibrium point of that Hamiltonian system is not necessarily hyperbolic. Indeed, the local solvability of the problem of disturbance attenuation for a nonlinear plant has been fully understood only for those systems whose first approximation is such that the corresponding problem is known to have a solution. Therefore, one objective research is to analyze the existence of solutions of the problem of disturbance attenuation for truly nonlinear systems, e.g. systems which may only be critically stabilizable, or which are detectable but not in the first approximation.

Task 6: Disturbance attenuation in singular cases, with application to robust stabilization.

Task 7: Derive necessary and sufficient conditions for the absence of classical blow-ups and shock solutions to the finite time HJI PDE's for H-infinity Control Problems in terms of a geometric Lagrangian interpretation of the associated Riccati PDE's.

Our next research task focuses on how nonlinear feedback laws also offer the potential to take advantage of genuinely nonlinear effects. One example is the incorporation of higher order nonlinear terms in feedback laws in order to shape the steady state response of nonlinear systems by shaping the flow on attractive, inertial or center manifolds. This has been used in the literature to asymptotically stabilize critically stable systems which cannot be stabilized by linear feedback laws. Other examples include the use of nonlinear feedback gains to produce or to attenuate the effects of limit cycles.

Task 8: A systematic investigation of the use of nonlinear control laws to take advantage of nonlinear phenomena.

There are of course many situations of interest, e.g. tracking and regulation, where the system output (e.g., position or tracking error) cannot be treated, or redefined, as a design

parameter but rather needs to be controlled to a given value. In this case, passivity constructions cannot be used directly if the relative degree of the system output is higher, as is typically the case for the position variables for a mechanical system. In such cases we would still like to take advantage of any compact attractors in, or Lyapunov stability of, the zero dynamics determined by the desired output constraint for the purpose of designing a locally asymptotically stabilizing law for a compact attractor. Accordingly, another of our explicit research tasks focuses on the development of a methodology for local feedback stabilization about an attractor.

Task 9: The development of systematic feedback design methodologies for semi-global and global stabilization about compact attractors.

Another research goal for lumped nonlinear control systems, is the development of (topological) necessary conditions similar in spirit to the well-known conditions for equilibrium.

Task 10: The development of systematic feedback design methodologies for local feedback stabilization about compact attractors.

In the interest of obtaining fixed bounds on omega-limits of closed-loop trajectories or of ensuring boundedness of responses to small disturbances, it is desirable to consider feedback stabilization about a compact attractor. Our next explicit research task addresses the need to establish criteria for the existence of compact attractors.

Task 11: The development of verifiable necessary conditions for the existence of asymptotically stable, or asymptotically stabilizable, compact attractors for broad classes on non-linear control systems.

2.2.2 Control of Nonlinear Distributed Parameter Systems

One of the long term goals of the proposed research effort is the development of a systematic methodology for the design of feedback control laws capable of achieving a variety of desired performance objectives for important classes of nonlinear distributed parameter systems. Motivated in part by problems of flow control and combustion control, where nonlinear effects actually can improve mixing, there is growing current interest in feedback design for nonlinear distributed parameter systems.

Accordingly, a considerable effort on our part is being applied to the development of geometric methods for feedback stabilization and for input/output stabilization based directly on the control of a nonlinear distributed parameter model, not upon a lumped approximation. Just as in the distributed parameter systems literature, recent successes in the control theory of lumped nonlinear systems often depended on finding the nonlinear enhancements of finite dimensional linear tools and the exploitation of nonlinear effects that do not have lumped linear counterparts. Indeed, there are several important features of boundary control for distributed parameter systems which are not inherited by finite dimensional approximations

- for example, the availability of point sensors and actuators. Remarkably, these features in fact make it easier to translate certain of the recent nonlinear concepts from the lumped to the distributed parameter case. Indeed, motivated by recent advances in lumped nonlinear control theory we have based our approach to the design of stabilizing feedback control laws on a combination of tools, specifically with center manifold methods and with zero dynamics, drawn from geometry and nonlinear dynamics.

Task 1: The development of systematic feedback design methodologies for the stabilization of minimum phase nonlinear distributed parameter systems. In particular, the development of the concept and properties of zero dynamics for linear and nonlinear distributed parameter systems, as well as a feedback design methodology based on asymptotic properties of the zero dynamics.

Our experience with the application of these geometric tools to the solution of the nonlinear regulator problem for lumped nonlinear systems indicates that we will also be able to make progress on the important problem of shaping the steady state response of nonlinear distributed parameter systems. It is worth noting that the active control in many flow control experiments in the control of instabilities in the unsteady separated shear layer is based on a priori harmonic forcing, while in nonlinear systems with resonance it is known that simple harmonic forcing will not necessarily produce the desired response.

Task 2: The investigation of the steady state behavior of certain classes of controlled nonlinear DPS modeling hydrodynamic phenomena through the derivation of a principal asymptotic phase for nonlinear zero dynamics. This involves, inter alia, analysis and feedback design for higher dimensional Burgers' systems and two-dimensional Navier Stokes systems on bounded domains.

Task 3: The determination of conditions for I/O stability and for finite square integrable gain of certain classes of controlled linear and nonlinear DPS modeling hydrodynamic phenomena, including one and higher dimensional Burgers' systems and two dimensional Navier-Stokes systems on bounded domains. Also, an investigation of the relationship between asymptotic and I/O stability nonlinear distributed parameter systems.

Our next research task is concerned with the development of feedback design methods for problems of asymptotic tracking, disturbance rejection and output regulation for nonlinear distributed parameter systems.

Since the same center manifold methods underlie nonlinear regulator theory for finite dimensional, neutrally stable exogenous signal generators and are independent of the particular technique used to design a stabilizing feedback law, we expect to be able to derive more general conditions, not just for harmonic forcing, but for asymptotic tracking and disturbance attenuation for exponentially stable systems.

Task 4: The development of a theory of robust output regulation for distributed parameter systems and its application to problems of asymptotic tracking and disturbance attenua-

tion. In particular, this will include the development of methods for output regulation for distributed parameter systems and the analysis of steady state response of stable nonlinear systems to signals produced by lumped nonlinear exogenous systems.

Our final research tasks in the control of distributed parameter are concerned with the impact of feedback control parameters on the structure of attractors and inertial manifolds for nonlinear distributed parameter systems. We are also interested in the problem of feedback equivalence to dissipative dynamical systems, and hence to a distributed parameter system possessing a finite dimensional inertial manifold and attractor, and its relationship to the more system theoretic notions of dissipation for passive systems.

Task 5: To develop the concept of passivity for nonlinear distributed parameter systems with inputs and outputs. We also intend to relate this to the more familiar concept of dissipativity for autonomous nonlinear evolution equations.

2.3 Analytical and Computational Methods for Nonlinear Feedback Design AFOSR Grant No. F49620-94-1-0438 Christopher I. Byrnes and Alberto Isidori, PI

Feedback controller design for both lumped and distributed parameter systems leads to the consideration of nonlinear partial differential equations and therefore to analytical questions such as existence, uniqueness, regularity, the formation of shocks, stability, of solutions to these equations. In this research effort we address several of these basic analytical problems with the goal of analyzing the behavior of feedback control strategies to shape the response of control systems. We shall also pursue the development of the analysis of the feedback design methodology for sampled nonlinear systems and for discrete-time, lumped nonlinear systems.

2.3.1 Nonlinear PDE's arising in lumped nonlinear control design

In recent years, it has been discovered that an impressive array of feedback design problems for nonlinear control systems can be solved in terms of the solution of certain systems of nonlinear PDE's. This is very close in spirit to the situation for linear control system design, where it is now well-known that a variety of basic design problems can be solved in terms of a hierarchy of matrix equations: the quadratic (nonlinear) Riccati equation, the Sylvestor equation and its more simple form, the Lyapunov equation. Basic and important problems of nonlinear control theory, such as feedback linearization, output regulation, optimal control and H-infinity control, are all now known to be solvable if, and only if, certain PDE's are solvable.

The most well-known example is the Hamilton-Jacobi-Bellman (HJB) PDE for optimal control, which is one nonlinear form of the matrix Riccati equation. A geometric existence

theory for the HJB PDE can also be reformulated using a Riccati PDE, for which Lagrangian analysis provides a geometric development of the notion of generalized functions and weak solutions. This also holds for the H-infinity approach to robust nonlinear control. It is necessary to solve a Hamilton-Jacobi-Isaacs partial differential equation (JHI PDE), which is highly nonlinear when the plant dynamic system is not affine in the inputs.

Another example is what is now known as the FBI (Francis-Byrnes-Isidori) partial differential equation for the problem of output regulation, which in the linear case reduces to the Sylvester equation discovered by Francis. In the nonlinear setting, this has been applied to some research problems of direct interest to the DoD. Finally, we remark that the Zubov equation represents a simpler form of an FBI equation, reducing to a Lyapunov equation in the linear case.

In general, there remain many open problems concerning the existence and nature of nonlinear control laws which are obtained from the off-line solution of a nonlinear PDE. For reasons of developing a systematic design methodology based on the off-line solution of the nonlinear enhancement of the Riccati, Sylvester, and Lyapunov equations, we propose an analysis of two of the main open questions. First, there is comparatively little known concerning the absence of classical blow-ups and of shock waves for solutions to the HJB, the HJI or to the Riccati PDE.

Task 1: Derive necessary and sufficient conditions for the absence of classical blow-ups and shock solutions to both the finite time and steady state HJB and Riccati PDE's in terms of a geometric Lagrangian interpretation of the associated Riccati PDE's.. To also relate these conditions to system theoretic properties or variational concepts, such as existence and uniqueness of optimal solutions.

Task 2: While Task 1 of Section 2.2.1 focuses on optimization problems with a convex Lagrangian functional, there are several important problems in modern control systems which involve steady state optimization with nonconvex Lagrangians. One such problem is the development of an enhancement of the Kalman-Yacubovitch-Popov Lemma for nonlinear passive systems where, motivated by nonlinear circuit theory, the Lagrangian is defined as the power of the input-output. In this case the nonconvex Hamilton-Jacobi partial differential inequality is precisely the Kalman-Yacubovitch-Popov Lemma. Since passivity underlies all known global stability mechanisms, the solution of this nonconvex problem would lead to new feedback design methodologies for stabilizing nonlinear control systems. Another well-known problem involving nonconvex optimization is H-infinity control theory, where the appropriate min-max Lagrangian represents the attenuation of the effect of a disturbance on a controlled output.

This component of our proposed research in lumped nonlinear control is therefore related to the phase portrait of a nonstandard Riccati ODE, also arising in linear H-infinity control. For many applications, the heart of the nonlinear, or indefinite, feedback control design problem is to design a feedback law which minimizes either the applied and resulting energy

of a system, or the attenuation factor relating the energy, or power, of the disturbance to that of the output to be controlled. It is known, for local problems in a steady state formulation with knowledge of the system state, that existence of solutions is equivalent to existence of solutions to the solvability of the corresponding steady state Riccati equation for the linearized problem. Some sufficient conditions, e.g. no imaginary axis transmission zeros, exist for solvability of the matrix Riccati equation with appropriate semidefiniteness. However, in the indefinite case, necessary and sufficient conditions, preferably stated in systems theoretic terms, are still unavailable.

Of paramount importance is the determination of conditions, stated in terms of easily checked system-theoretic properties, for the nonexistence of "finite escape time" of solutions, for a desired attenuation factor "gamma" and a desired time horizon. This is particularly important since gamma represents the attenuation factor for the perturbations affecting the closed loop system. Of course, for gamma sufficiently large the sign-indefinite nature of the quadratic term disappears and existence may be studied in terms of the Riccati equation the corresponding LQ design problem. While sufficient system-theoretic conditions for the solvability of the LQ Riccati equations has been known since 1960, a complete phase portrait for the matrix Riccati equation was developed only recently by Shaman, based on earlier work of Hermann and Martin in which the phase space of the Riccati equation was compactified in a Lagrangian Grassmann manifold. This "compactification" of the Riccati equation greatly simplifies the description of the phase portrait by replacing "finite escape time" by an intersection criterion.

The indefiniteness of the matrix Riccati equation arising in H-infinity control is an expression of the nonconvexity of the Lagrangian functional in the game theoretic formulation of the problem. Similarly, progress on indefinite matrix Riccati equations arising in control and estimation will lead to new feedback design methodologies for passive systems. Finally, indefinite matrix Riccati equations also arise in dual problems such as filtering and estimation, when the covariance data is incomplete or not positive definite. Many important problems in signal processing and speech synthesis depend on being able to solve such indefinite equations.

Task 3: Phase portrait of indefinite Riccati equations. The development of a complete phase portrait of the indefinite Riccati equation as a dynamical system evolving on the Lagrangian Grassmannian. Using this complete phase portrait, the formulation of system theoretic, necessary and sufficient conditions for the existence of solutions to the Riccati equation of H-infinity control, of passive systems, and of filtering and covariance extension.

It is important to be able to apply these feedback design methodologies to the sample-and-hold control schemes which will actually be used in practice. This will involve the following research task as an important component of our overall effort.

Task 4: To extend these new feedback design methodologies for lumped, continuous-time nonlinear systems to lumped, discrete-time nonlinear systems.

Global properties of controlled nonlinear distributed parameter systems. Our research objectives for distributed parameter systems include the further development of a root-locus design methodology for linear, parabolic distributed parameter systems as well as the extension of the concept to nonlinear distributed parameter systems via the notion of transmission zeros and zero dynamics. A second major research objective involves the development of a design methodology capable of shaping the response of distributed parameter systems.

In particular, we are interested in the problem of output regulation for boundary controlled linear distributed parameter systems. Our preliminary calculations indicate that some of the special physical features of distributed parameter systems, such as boundary control or sensors, which do not manifest themselves in finite dimensional approximations are extremely useful in the design of feedback laws for stabilization and control. For a class of linear parabolic systems we have been able to analyze the steady state response of an asymptotically stable system, and based on this work, we expect to be able to solve the problem of output regulation for this class of control systems and for a linear exogenous system.

Task 5: Develop a theory of output regulation for linear distributed parameter systems with application to problems of asymptotic tracking and disturbance attenuation. We are particularly interested in developing geometric methods for the analysis of steady state response of stable linear systems to signals produced by lumped exogenous systems.

Our longer range goals also include problems of global stabilization and output regulation for boundary controlled nonlinear distributed parameter systems of hydrodynamic type. We expect our analysis of linear distributed parameter control to provide some insight into particular local nonlinear problems, but for nonlinear problems one needs to develop additional tools to understand global behavior. An important prerequisite is the analysis of the long time dynamics of such systems.

Some of the most outstanding problems in mathematical fluid dynamics are concerned with questions of global in time existence and uniqueness of solutions to hydrodynamic problems such as the Navier-Stokes system. This type of difficulty is even more evident for problems in which the control action takes place through the boundary since the form of the boundary conditions plays a critical role in determining the properties of the dynamics. For example, for Dirichlet or periodic boundary conditions for Burgers' equation, the nonlinear terms disappear in the energy balance relation for the weak solution, but for any other type of physical boundary conditions the nonlinear terms persist in this relation and the presence of these terms greatly complicate the analysis of the global in time existence of the solutions.

Task 3: A rigorous mathematical investigation of the global in time existence, uniqueness and regularity of solutions to boundary controlled systems of hydrodynamic type, i.e., systems containing convection as well as reaction diffusion terms.

2.4 Multidisciplinary Rotorcraft Analysis and Simulation DoD Grant No. DAAH04-94-6-0351 David Peters, PI

During the past 20 years, rotor dynamicists have developed a number of widely-used tools to analyze the coupled structural dynamics and aerodynamics of rotor-body systems. These include trim techniques (shooting, harmonic balance, and auto-pilots), Floquet theory, multi-blade coordinates, and dynamic inflow. All of these, however, are more or less predicated on constant RPM and on periodic coefficient of known period. Recent implementations of SGCHAS, however, have revealed that inclusion of the engine-drive-train dynamics in a unified model makes the RMP time varying and often unknown a priori. Thus, rotor stability, torsional stability, and aircraft stability and control cannot be handled in conventional ways.

We have undertaken the investigation of the modifications of present methods and the introduction of new methods that would be necessary to do a multidisciplinary rotorcraft analysis which would include engine, control system, drive train, rotor, and wake dynamics with the resultant unsteady RPM.

2.5 Augmentation of Trim Analysis Procedures, DoD Grant No. DAAH 04-96-1-0111 David Peters, PI

During the past 20 years rotor dynamicists have developed a number of widely-used tools to analyze the coupled structural dynamics and aerodynamics of rotor-body systems. These include trim techniques (shooting, harmonic balance, and auto-pilots), Floquet theory, multi-blade coordinates, and dynamic inflow. All of these, however, are more or less predicated on constant RPM and on periodic coefficients of known period. Recent implementations of SGCHAS, however, have revealed that inclusion of the engine-drive-train dynamics in a unified model makes the RPM time varying and often unknown a priori. Thus rotor stability, torsional stability, and aircraft stability and control cannot be handled in conventional ways.

In the present research, we are studying modifications of present methods and the introduction of new methods in order to do a multidisciplinary rotor analysis that includes engine, control system, drive train, rotor, and wake dynamics with the resultant unsteady RPM. We have made substantial progress including the development of a general theory of rotorcraft trim that encompasses unsteady and unknown RPM.

Two new and exciting developments have occurred that were not anticipated in the original proposal but which impact the research. One is the development of Fast Floquet Theory that allows a Q-fold savings in CPU time (where Q is the number of blades). The second is the introduction of the concept of a Discrete Auto-pilot to effect an efficient trim.

2.6 Novel Approaches to the Stability Analysis of Complex Rotorcraft Systems

Stability analysis is an important concern in rotorcraft analysis, and yet only a limited number of tools are available for this purpose. More often than not, a linearized stability analysis is performed: the spatially discretized governing differential equations of motion are linearized to yield ordinary differential equations with either constant or periodic coefficients.

When the coefficients are constant, stability information can be readily obtained from the real part of the characteristic exponent: a positive real root corresponds to an exponentially growing amplitude of the motion. When the coefficients are time dependent or periodic, as often occurs in rotorcraft problems, Floquet theory must be used. This approach relies on the analysis of the transfer matrix which characterizes the evolution of arbitrary initial conditions over one period. When this evolution involves a growing amplitude of the response (detected by a characteristic value of the transition matrix larger than unity) and instability is detected. This approach has been widely used in rotorcraft problems.

Unfortunately, the Floquet theory presents several drawbacks: First, it is limited to systems involving a relatively small number of degrees of freedom. Indeed, as the number of degrees of freedom increases, the cost of generation the transition matrix become staggering, and numerical problems are encountered due to the increased ill-conditioning of the transition matrix. Furthermore, this approach inherently deals with linearized stability, i.e. the stability of small perturbations about a periodic solution and therefore requires the cumbersome task of a rigorous linearization of the governing equations.

As a result, stability analysis is often performed on simplified models involving the strict minimum number of degrees of freedom necessary to capture the physical phenomenon that causes the instability.

Comprehensive rotorcraft codes based on more and more detailed structural dynamic models and increasingly sophisticated aerodynamic and wake models are receiving increased attention in an effort to improve the accuracy of the analysis. The very nature of these comprehensive analyses is such that a very large number of degrees of freedom is involved. Finite element analyses of the structural dynamic behavior of rotor systems involve thousands of degrees of freedom, whereas tens or hundreds of thousand of degrees of freedom are not uncommon in computational fluid dynamics analysis. It is clear that the classical methods of stability analysis cannot be applied to such models. There is also a need to address the question of nonlinear stability, such as the limit cycle phenomenon, for instance. Indeed, recognizing the importance of nonlinear effects is a driving force in the development of more accurate comprehensive analysis tools.

Comprehensive analyses merely simulate, as accurately as possible, the evolution of the physical system. Stability information can be obtained by integration over a long period of time and observing the resulting decreasing (or increasing) amplitude of the response. Two drawbacks are associated with this approach. First, it can be rather subjective: is an

increasing amplitude of the response the result of an instability, or is it merely a transient behavior that would disappear had a longer period of integration been considered? Second, a physical instability can be masked by the numerical damping which is indispensable for the stability of the numerical integration procedure.

Two novel approaches are proposed for the stability analysis of complex rotorcraft systems, the *energy map* approach and the *partial state Floquet* approach.

2.7 Finite-State Dynamic Inflow Models for Maneuvering and Inground Effect Flight Conditions Center No. 10/24-6-R8943-0A0, Cooperative Agreement No. NCC 2-945 David A. Peters and J. V. R. Prasad, PI

The generalized dynamic inflow models for both hover and forward flight conditions developed during the late 80's are routinely used in the helicopter industry, while capable of accurately describing the time-varying inflow distribution at the rotor, they can easily be combined with other aeroelastic and flight dynamic models for stability and control analysis and for real-time simulation. However, since effects due to wake distortion such as curvature, compression and dynamics are not taken into account in those models, their applicability to maneuvering and in-ground effect flight conditions is somewhat limited. In fact, current simulation codes which are based on existing inflow models do not accurately capture the off-axis response characteristics, e.g., roll response to longitudinal cyclic input. Almost all simulation codes predict that off-axis response behavior is important for development and evaluation of model decoupling flight control laws and for effective pilot training using simulation. Recent results from Technion have identified that the effect of vortex spacing and wake curvature during maneuvering flight can explain the correct off-axis response behavior during maneuvering flight.

Also, existing dynamic inflow models use empirical factors to account for ground effect and cannot accurately predict in-ground effect inflow distribution for cases such as inclined ground effect (e.g., sloped landings), partial ground effect (e.g., hovering over edge of a building, over edge of a ship deck) and dynamic ground effect (e.g., hovering over a pitching and rolling ship deck). Thus there is a definite need for extending the applicability of current dynamic inflow models to maneuvering and in-ground effect flight conditions.

The objectives of this research are to investigate several types of wake distortion (including wake skew, wake curvature, wake compression, and wake dynamics) to see if these can be utilized in a simple way to account for correct inflow distribution over the rotor during maneuvering and in-ground effect flight conditions and to evaluate resulting models through extensive in-house simulations and through high fidelity simulations and pilot evaluations using simulation facilities at industry and government labs (see MOA's).

The approach for maneuvering flight is to compare momentum theory, vortex-tube theory, and vortex lattice theory in order to determine the effect of various sources of off-axis coupling

that arise from inflow dynamics. This will allow such coupling effects to be included in conventional inflow and dynamic wake formulations. Such analyses are easily modified to include the correct effect. Comparisons with flight tests and wind tunnel data will be used to identify the most appropriate model. One of the most difficult areas to model will be hover and low-speed flight. Here, some type of free-wake may be needed to identify the behavior, but finite state method will be used to produce the identified behavior in simulations. The approach for ground effect is to represent the rotor and an image rotor (for satisfying the flow boundary conditions at the ground surface) as distributed pressure discontinuities and use superposition principle in the development of Inground effect dynamic inflow models. In order for the superposition to be strictly valid, the free stream flow velocity needs to be the same for the rotor as well as for its image, which is true only for forward flight conditions. For hovering flight, the approach needs to be modified by using something other than an image rotor for satisfying the flow velocity boundary condition at the ground plane. Initial results obtained at Georgia Tech using this approach look promising. The methodology is being fully developed and will be extended to model inclined, partial and dynamic ground effect cases. The dynamic inflow models developed in this research will be validated using the UH-60L, CH-53E and the Comanche helicopter flight test data as well as wind tunnel data.

The proposed research is being performed with collaboration from the flying qualities groups at Boeing and Sikorsky and with researchers from NAWC and NASA Ames. In addition to problem definition and use of flight test data, the industry and government collaborations permitted the use of high fidelity simulation facilities for validation and pilot evaluations of dynamic inflow models developed in this research.

Development of models and in-house simulation validations through comparisons with existing flight test and wind tunnel data has been performed during the first three years to arrive at the correct dynamic inflow model. Also, research cooperation with Boeing Sikorsky, NAWC and NASA initiated and validations using their high fidelity simulation facilities have been conducted.